Remarks about decay widths of two-body nonleptonic and semileptonic B decays

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Abstract

We present a brief discussion about expressions of decay widths of exclusive nonleptonic and semileptonic B decays at tree level including $l = 0$ and $l = 1$ mesons in final state. Our analysis is carried out assuming factorization hypothesis and using parametrizations of hadronic matrix elements given in WSB and ISGW quark models. Special interest is focused on dynamics of these processes and several important ratios between decay widths to determine form factors and decay constants are given.

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1 Introduction

Exclusive semileptonic and nonleptonic B decays offer a good scenario for studying, at theoretical and experimental levels, CP violation, precise determination of Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, new physics beyond the Standard Model, QCD and electroweak penguin effects, production of orbitally excited mesons, etc, which are topics of great interest of research in particle physics of this decade [1].

The purpose of this paper is to perform an overview about expressions of decay widths of exclusive nonleptonic and semileptonic B decays at tree level including $l = 0$ and $l = 1$ mesons in final state. For $l = 0$, we have considered pseudoscalar (P) and vector (V) mesons, and for p-wave mesons we have included scalar (S), axial (A) and tensor (T) mesons (see Table I). We present an interesting summary and a panoramic analysis about expressions of decay widths of nonleptonic $B \to P_1P_2, VP, V_1V_2, AP, AV, A_1A_2, PS, SS, SV, SA, TP, TV, TS, TA$ decays[1], and differential decay rates of semileptonic $B \to (P, V, S, A, T)l\nu$ decays. We have assumed factorization hypothesis and used parametrizations of hadronic matrix element $\langle M|J_\mu|B\rangle$ given in relativistic WSB [2] and nonrelativistic ISGW [3] quark models.

Our aim is to give a general point of view about $\Gamma(B \to M_1M_2)$ and $d\Gamma(B \to Ml\nu)/dt$, illustrating the dynamics of these decays and pointing out that some relations among these expressions and form factors that participate in the parametrization of hadronic matrix element $\langle M|J_\mu|B\rangle$, which is common in both processes, could be experimental tests. For the sake of simplicity we have only considered tree level color-allowed external W-emission nonleptonic B decays, i.e. the so called *class-I* decays in the literature [4].

The paper is organized as follows: Section 2 contains expressions for $\Gamma(B \to M_1M_2)$ and $d\Gamma(B \to Ml\nu)/dt$ and a discussion about them. In section 3, we analyze vector and axial contributions of weak interaction to $B \to (P, V, S, A, T)l\nu$ decays assuming a meson dominance model. In section 4, we quote some important relations among decay widths, which allow us to determine some form factors and decay constants. Finally, in section 5 we present some concluding remarks.

2 $\Gamma(B \to M_1M_2)$ and $d\Gamma(B \to Ml\nu)/dt$

In this section we summarize expressions at tree level of differential decay rates of $B \to (P, V, S, A, T)l\nu_l$ (see Table II) and decay widths of $B(b\bar{q}') \to M_1(q\bar{q}')M_2(q_i\bar{q}_j)$, where $M_{1,2}$ can be a pseudoscalar (P), a vector (V), a scalar (S), an axial-vector (A) or a tensor (T) meson (see Table III)[2]. For nonleptonic two-body B decays we have used the notation $B \to M_1, M_2$ [6] to mean that M_2 meson is factorized out under factorization approximation, i.e., M_2 arises from vacuum. For $B \to P$ and $B \to V$ transitions we have used the parametrizations given in relativistic WSB quark model [2] and for $B \to S$, $B \to A$ and $B \to T$ transitions the ones given in nonrelativistic ISGW quark model [3] because it is the only quark model that had calculated these transitions. We have considered both $^1P_1$ and $^3P_1$ axial-vector mesons.

[1]For definiteness we use a B-meson in our notation, but the results are quite general. They apply equally well to D-mesons, or even to pseudoscalar mesons.

[2]The Ref. [5] also gives a summary about differential decay rates of $\Gamma(B \to P(V)l\nu_l)$ and decay rates of $B \to P_1P_2, PV, VP, V_1V_2$.

Table I. Spectroscopic notation for $l = 0$ and $l = 1$ mesons. l is the orbital angular momentum, s is the spin, and J is the total angular momentum. P and C are parity and charge conjugate operators, respectively.

l	s	J	${}^{2s+1}L_J$	J^{PC}	Meson
$l = 0$	$s = 0$	$J = 0$	${}^1S_0$	0^{-+}	Pseudoscalar (P)
	$s = 1$	$J = 1$	${}^3S_1$	1^{--}	Vector (V)
$l = 1$	$s = 0$	$J = 1$	${}^1P_1$	1^{+-}	Axial-vector $\big(A({}^1P_1)\big)$
	$s = 1$	$J = 0$	${}^3P_0$	0^{++}	Scalar (S)
		$J = 1$	${}^3P_1$	1^{++}	Axial-vector $\big(A({}^3P_1)\big)$
		$J = 2$	${}^3P_2$	2^{++}	Tensor (T)

In second column of Table II, we list $d\Gamma/dt$ for exclusive semileptonic decays $B \to Ml\nu$ including all $l = 0$ and $l = 1$ mesons (i.e., pseudoscalar, vector, axial-vector, scalar and tensor mesons) in final state, in function of form factors (defined in WSB and ISGW quark models) and powers of $\lambda = \lambda(m_B^2, m_M^2, t)$, where $\lambda = \lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ is the Euler function and $t = (p_B - p_M)^2$ is the momentum transfer. In first row we give this expression for $B \to Pl\nu$, where P is a pseudoscalar meson [7]; in second row we present $d\Gamma/dt$ for $B \to Vl\nu$, where V is a vector meson, in function of the form factors $A_{1,2}(t)$ and $V(t)$, and also in function of helicity form factors which are defined in appendix B [8]; in third row we show differential decay width for $B \to A({}^1P_1)l\nu$, where $A({}^1P_1)$ is an axial-vector meson [9] using the parametrization for transition $B \to A$ given in ISGW quark model; in fourth row we display $d\Gamma/dt$ for $B \to Sl\nu$. We have obtained this expression from $d\Gamma(B \to Pl\nu)/dt$ in two steps: first we change the form factors given in WSB quark model by the ones given in ISGW quark model using the formulae showed in appendix A, and second changing the form factors $f_{+,-}$ by $u_{+,-}$, which are used in the parametrization of $\langle S|J_\mu|B\rangle$; in fifth row we present the differential decay rate for $B \to A({}^3P_1)l\nu$, where $A({}^3P_1)$ is an axial-vector meson. We have obtained it from the similar expression displayed in third row only changing form factors. Finally, in last row we show $d\Gamma/dt$ for $B \to Tl\nu$, where T is a tensor meson [9].

In Table II, $F_1(t)$ and $F_0(t)$ are monopolar form factors [2], $u_\pm$ are form factors defined in appendix B of Ref. [3]. The common factor ζ and functions $A(t)$, $B(t)$, $\mathcal{G}(t)$, $\varphi(t)$, $\rho(t)$, $\theta(t)$, $\alpha(t)$, $\beta(t)$ and $\gamma(t)$ are defined by:

$$\zeta = \frac{G_F^2 |V_{qb}|^2}{192\pi^3 m_B^3}, \tag{1}$$

$$A(t) = \left(\frac{t - m_l^2}{t}\right)^2 \left(\frac{2t + m_l^2}{2t}\right), \tag{2}$$

$$B(t) = \frac{3}{2} m_l^2 \left(\frac{t - m_l^2}{t}\right)^2 \frac{(m_B^2 - m_P^2)^2}{t}, \tag{3}$$

Table II. Differential decay widths of $B \to (P, V, S, A, T)l\nu_l$.

Decay	$d\Gamma(B \to Ml\nu_l)/dt$
$B \to Pl\nu_l$	$\zeta\left[A(t)\lvert F_1(t)\rvert^2\lambda^{3/2} + B(t)\lvert F_0(t)\rvert^2\lambda^{1/2}\right]$
$B \to Vl\nu_l$	$\zeta\mathcal{G}(t)$
	$\zeta t\lambda^{1/2}\left[\lvert H_+(t)\rvert^2 + \lvert H_-(t)\rvert^2 + \lvert H_0(t)\rvert^2\right]$
$B \to A(^1P_1)l\nu_l$	$\zeta\left\{\varphi(t)\lambda^{5/2} + \rho(t)\lambda^{3/2} + \theta(t)\lambda^{1/2}\right\}$
$B \to Sl\nu_l$	$\zeta\left[A(t)u_+^2\lambda^{3/2} + B(t)\frac{t^2}{(m_B^2-m_S^2)^2}(u_+ + u_-)^2\lambda^{1/2}\right]$
$B \to A(^3P_1)l\nu_l$	$\zeta\left\{\varphi(t)\lambda^{5/2} + \rho(t)\lambda^{3/2} + \theta(t)\lambda^{1/2}\right\}$
$B \to Tl\nu_l$	$\zeta\left\{\alpha(t)\lambda^{7/2} + \beta(t)\lambda^{5/2} + \gamma(t)\lambda^{3/2}\right\}$

$$\mathcal{G}(t) = \left[\frac{2t|V(t)|^2}{(m_B+m_V)^2} + \frac{(m_B+m_V)^2|A_1(t)|^2}{4m_V^2} - \frac{(m_B^2-m_V^2-t)A_1(t)A_2(t)}{2m_V^2}\right]\lambda^{3/2} + \frac{|A_2(t)|^2}{4m_V^2(m_B+m_V)^2}\lambda^{5/2} + 3t(m_B+m_V)^2|A_1(t)|^2\lambda^{1/2}, \tag{4}$$

$$\varphi(t) = \frac{s_+^2}{4m_A^2}, \tag{5}$$

$$\rho(t) = \frac{1}{4m_A^2}\left[r^2 + 8m_A^2 t v^2 + 2(m_B^2 - m_A^2 - t) r s_+\right], \tag{6}$$

$$\theta(t) = 3t\, r^2, \tag{7}$$

$$\alpha(t) = \frac{b_+^2}{24m_T^4}, \tag{8}$$

$$\beta(t) = \frac{1}{24m_T^4}\left[k^2 + 6m_T^2 t h^2 + 2(m_B^2 - m_T^2 - t) k b_+\right], \tag{9}$$

$$\gamma(t) = \frac{5tk^2}{12m_T^2}, \tag{10}$$

where G_F is the Fermi constant, m_B is the mass of B meson, $m_{V(A)}$ is the mass of vector(axial) meson, $m_{P(T)}$ is the mass of pseudoscalar(tensor) meson, m_l is the mass of lepton, $V(t)$ and $A_{1,2}(t)$ are monopolar form factors [2], $\varphi(t)$, $\rho(t)$ and $\theta(t)$ are quadratic functions of form factors s_+, r and v (which are defined in appendix B of ISGW model [3]) for $B \to A(^1P_1)l\nu$ and of form factors c_+, l and q (also given in appendix B of Ref. [3]) for $B \to A(^3P_1)l\nu$, respectively; $\alpha(t)$, $\beta(t)$ and $\gamma(t)$ are quadratic functions [9] of

form factors k, b_+ and h, which also are given in appendix B of [3].

Comparing parametrizations of $B \to P$ and $B \to S$ transitions given in sections 1 and 5, respectively, of appendix B of ISGW quark model [3] it is easy to relate expressions of differential decay rates for $B \to Pl\nu$ and $B \to Sl\nu$. In similar way, if we compare parametrizations of $B \to V$ and $B \to A(^3P_1)$ (or $B \to A(^1P_1)$) transitions given in sections 2 and 4 (or 6), respectively, of appendix B of ISGW quark model [3] it is straightforward to establish a connection between differential decay rates for $B \to Vl\nu$ and $B \to A(^3P_1)l\nu$ (or $B \to A(^1P_1)l\nu$). This is because the role of axial and vector currents is interchanged in both cases. In Ref [10] appears a brief discussion about it.

Let us discuss the dependence of $d\Gamma(B \to Ml\nu)/dt$ with the function λ (note that $|\overrightarrow{p}| = \lambda^{1/2}/2m_B$, where $\overrightarrow{p}$ is the three-momentum of meson M in the B meson rest frame): in general, $d\Gamma/dt \sim \lambda^{l+\frac{1}{2}}$, where l is the orbital angular momentum which is associated to the wave that particles can be coupled in final state. Demanding conservation of total angular momentum J and assuming a meson dominance model it can be found specific values for l in each exclusive semileptonic $B \to Ml\nu$ decay. Thus, in $B \to Pl\nu$ and $B \to Sl\nu$ particles are coupled to waves $l = 0$ and $l = 1$ in final state; in $B \to Vl\nu$ and $B \to Al\nu$ particles can be coupled to waves $l = 0$, $l = 1$ and $l = 2$ in final state; and in $B \to Tl\nu$ to waves 1, 2 and 3.

We display, in Table III, decay widths of exclusive W-external nonleptonic $B \to M_1, M_2$ decays at tree level (the so called *type-I* decays) assuming factorization hypothesis. We have considered all the mesons $l = 0$ and $l = 1$ for M_1 and M_2. As $\langle T|J_\mu|0\rangle = 0$ [11] we do not consider $B \to M, T$ decays. Again, for transitions $B \to P$ and $B \to V$ we use the relativistic WSB quark model [2] and the nonrelativistic ISGW quark model [3] for $B \to S$, $B \to A$ and $B \to T$ transitions. The WSB quark model only works with $B \to P$ and $B \to V$, i.e., with $l = 0$ mesons. For this reason we use both quark models. Let us mention that in Ref. [12] appears parametrization of $B \to A(^1P_1)$ transition which has the same structure of parametrization of $B \to V$ transition interchanging the role of vector and axial currents: $\langle A|A_\mu|B\rangle \leftrightarrow \langle V|V_\mu|B\rangle$ and $\langle A|V_\mu|B\rangle \leftrightarrow \langle V|A_\mu|B\rangle$. The expressions for $\Gamma(B \to P_1, P_2;\ P, V;\ V, P;\ V_1, V_2)$ are given in several references (see for example [2]); $\Gamma(B \to T, P;\ T, V)$ were reported in Ref. [13] and we have taken $\Gamma(B \to P, A;\ A, P;\ A, V;\ A, A)$ from Ref. [14].

In Table III, all form factors ($F_{1,0}$, A_0,$A_{1,2}$,$H_\pm$, H_0,f_+, r, and $s_\pm$) are evaluated in m_2^2 because the momentum transfer $t = (p_B - p_1)^2 = p_2^2 = m_2^2$. In similar way the function λ is $\lambda(m_B^2, m_1^2, m_2^2)$ for nonleptonic B decays. $\varphi(m_A^2)$, $\rho(m_A^2)$ and $\theta(m_A^2)$ has the same form of Eqs. (5), (6) and (7) just changing $r \to f$, $s_+ \to a_+$ and $v \to g$, which are the appropriate form factors for $\langle V|J_\mu|B\rangle$ transition in ISGW-model. We can also add to Table III expressions for $\Gamma(B \to P, A(^3P_1))$, $\Gamma(B \to A(^3P_1), P)$, $\Gamma(B \to A(^3P_1), V)$ and $\Gamma(B \to V, A(^3P_1))$ only changing the form factors $r \to l$, $s_\pm \to c_\pm$ and $v \to q$. The constant $\xi^{(M_2)}$ and the function $\mathcal{F}^{B\to T}$ are given for the following expressions:

$$\xi^{(M_2)} = \frac{G_F^2|V_{qb}|^2|V_{q_iq_j}|^2a_1^2f_{M_2}^2}{32\pi m_B^3}, \tag{11}$$

$$\mathcal{F}^{B\to T}(m_P^2) = k + (m_B^2 - m_T^2)b_+ + m_P^2 b_-, \tag{12}$$

where k, $b_\pm$ and h are form factors given in ISGW-model [3], evaluated at $t = m_P^2$,

Table III. Decay widths of $B \to P_1P_2, PV, V_1V_2, TP, TV, AP, AV$.

Decay	$\Gamma(B \to M_1, M_2)$
$B \to P_1, P_2$	$\xi^{(P_2)}(m_B^2 - m_1^2)^2 \lvert F_0(m_2^2)\rvert^2 \lambda^{1/2}$
$B \to P, V$	$\xi^{(V)} \lvert F_1(m_V^2)\rvert^2 \lambda^{3/2}$
$B \to V, P$	$\xi^{(P)} \lvert A_0(m_P^2)\rvert^2 \lambda^{3/2}$
$B \to V_1, V_2$	$\xi^{(V_2)} \mathcal{G}(t = m_{V_2}^2)$
	$\xi^{(V_2)} m_2^2 \lambda^{1/2} \left[\lvert H_+(m_{V_2}^2)\rvert^2 + \lvert H_-(m_{V_2}^2)\rvert^2 + \lvert H_0(m_{V_2}^2)\rvert^2\right]$
$B \to T, P$	$\xi^{(P)}(1/24m_T^4)\lvert \mathcal{F}^{B\to T}(m_P^2)\rvert^2 \lambda^{5/2}$
$B \to T, V$	$\xi^{(V)} \left[\alpha(m_V^2)\lambda^{7/2} + \beta(m_V^2)\lambda^{5/2} + \gamma(m_V^2)\lambda^{3/2}\right]$
$B \to P, A(^1P_1)$	$\xi^{(A)} \lvert f_+(m_A^2)\rvert^2 \lambda^{3/2}$
$B \to A(^1P_1), P$	$\xi^{(P)}/4m_A^2 \left\{r + s_+(m_B^2 - m_A^2) + s_- m_P^2\right\} \lambda^{3/2}$
$B \to A(^1P_1), V$	$\xi^{(V)} \left\{\varphi(m_V^2)\lambda^{5/2} + \rho(m_V^2)\lambda^{3/2} + \theta(m_V^2)\lambda^{1/2}\right\}$
$B \to V, A(^1P_1)$	$\xi^{(A)} \left\{\varphi(m_A^2)\lambda^{5/2} + \rho(m_A^2)\lambda^{3/2} + \theta(m_A^2)\lambda^{1/2}\right\}$

f_{M_2} is the decay constant of meson $M_2(q_i\bar{q}_j)$, a_1 is the QCD factor, and $|V_{qb}|$ and $|V_{q_iq_j}|$ are the appropriate CKM factors.

Note that in Table III it is straightforward to add decay rates for channels as $B \to AA$, $B \to SS$, $B \to SP$, $B \to SV$, $B \to AS$, $B \to TA$, and $B \to TS$ keeping in mind that the role of vector and axial currents of weak interaction is interchanged in $B \to S$ and $B \to P$, and in $B \to V$ and $B \to A$ transitions.

3 Contributions of vector and axial couplings

In this section we illustrate how can be coupled particles in final state in $B \to Ml\nu$ and $B \to M_1M_2$ to specific waves, and determine the quantum numbers of poles that appear in the monopolar form factors, assuming a meson dominance model. Moreover, it is possible to check which form factor is related with which wave. We show that these set of circumstances arise from vector and axial couplings of weak interaction. In order to explain this situation we consider the chain $B \to MM^* \to MW^* \to Ml\nu(q_1\bar{q}_2)$, where M^* is the pole and W^* is the off-shell intermediate boson of weak interaction. We can do it combining parity and total angular momentum conservations in $B \to MM^*$ strong process.

In Table IV, we show if specific waves that particles can be coupled in final state of semileptonic $B \to (P, V, S, A, T)l\nu$ decays come from vector or axial contributions. Axial-vector meson A can be $^1P_1$ or $^3P_1$. In order to explain the respective analysis to each channel we must keep in mind that the off-shell W boson can has spin 0 or 1. Thus, in vectorial coupling there are two possibilities: $S_W = 0$ with $P_W = +1$, and $S_W = 1$

with $P_W = -1$ (S_W and P_W denote spin and parity of the off-shell W boson, respectively). In similar way, in axial coupling there are two options: $S_W = 0$ with $P_W = -1$ and $S_W = 1$ with $P_W = +1$. Thus, we have four situations: 0^+, 1^-, 0^- and 1^+. They are displayed in second column of Table IV. Demanding both total angular momentum conservation ($J_{initial} = J_{final}$) and parity conservation ($P_{initial} = P_{final}$) of the process $B \to MM^*$, where M^* is the pole, we found contributions showed in Table IV. A similar analysis can be performed for nonleptonic $B \to M_1M_2$ decays.

Table IV. Vector and axial contributions to semileptonic $B \to (P, V, S, A, T)l\nu$ decays.

Contribution	J^P of W^*	$B \to Pl\nu$	$B \to Vl\nu$	$B \to Sl\nu$	$B \to Al\nu$	$B \to Tl\nu$
Vector	0^+	$l = 0$			$l = 1$	
	1^-	$l = 1$	$l = 1$		$l = 0$, $l = 2$	$l = 2$
Axial	0^-		$l = 1$	$l = 0$		$l = 2$
	1^+		$l = 0$, $l = 2$	$l = 1$	$l = 1$	$l = 1$, $l = 3$

In Table V, we show the respective form factors with the corresponding poles in $B \to Pl\nu$ and $B \to Vl\nu$ decays. In second column we list the quantum numbers J^P of poles, which are the same J^P options for the off-shell W boson (see second column in Table IV). In this case, we must check form factors that appear in parametrization of hadronic matrix elements $\langle M|V_\mu|B\rangle$ and $\langle M|A_\mu|B\rangle$ for $M = P, V$. Following this idea, we can extrapolate quantum numbers of poles for $B \to Ml\nu$ where M is a p-wave (or orbitally excited) meson: for $B \to Sl\nu$ the poles are 0^- and 1^+; for $B \to Al\nu$, where axial-vector meson A can be ${}^1P_1$ or ${}^3P_1$, are 0^+, 1^- and 1^+; and for $B \to Tl\nu$ the poles are 1^-, 0^- and 1^+. This result is important if we are interested in performing a quark model with monopolar form factors for $B \to S$, $B \to A$ and $B \to T$ transitions, i.e., considering $l = 1$ mesons in final state.

As an example, we illustrate from Tables IV and V the situation about $B \to Pl\nu$: this decay has two contributions $l = 0$ and $l = 1$ (see the exponents of λ in first row of Table II) which arise from vector coupling of weak current. The respective poles have quantum numbers 0^+ and 1^-, and form factors are F_0 and F_1, respectively (see appendix B).

Table V. Form factors related to vector and axial contributions of weak interaction to semileptonic $B \to (P, V)l\nu$ decays.

Contribution	J^P of Pole	$B \to Pl\nu$	$B \to Vl\nu$
Vector	0^+	$F_0(t)$	
	1^-	$F_1(t)$	$V(t)$
Axial	0^-		$A_0(t)$
	1^+		$A_1(t)$, $A_2(t)$, $A_3(t)$

4 Useful ratios

In this section we establish some ratios using expressions for $d\Gamma(B \to Ml\nu)/dt$ and $\Gamma(B \to M_1 M_2)$, which are displayed in Tables II and III, respectively.

4.1 Let us considerer the *type-I* $B \to P^{+,0}, V^-$ and $B \to V^{+,0}, P^-$ decays, where $P^{+,0}$ and $V^{+,0}$, and, V^- and P^- have the same quark content; i.e., the CKM factors are common in both processes. Moreover, we also assume that phase spaces are equal. From Table III, we obtain the following ratio:

$$\frac{\Gamma(B \to P, V)}{\Gamma(B \to V, P)} = \left(\frac{f_V}{f_P}\right)^2 \left[\frac{F_1^{B\to P}(0)}{A_0^{B\to V}(0)}\right]^2 \left[\frac{1 - m_P^2/m_{0^-}^2}{1 - m_V^2/m_{1^-}^2}\right]^2, \tag{13}$$

where we have used expressions of monopolar form factors F_1 and A_0 (see appendix B). As an application of Eq. (13) (following the presentation of Ref. [15]) we can consider exclusive $\overline{B^0} \to \pi^+, \rho^-$ and $\overline{B^0} \to \rho^+, \pi^-$ decays. Ref. [16] reports $\mathcal{B}(\overline{B^0} \to \rho^\pm \pi^\mp) = (2.28 \pm 0.25)\text{x}10^{-5}$ for the sum of the charge states or particle/antiparticle states indicated. We assume that these branching ratios are approximately equal[3]. Taking numerical values of masses of respective mesons reported in [16] and pole masses given in Ref. [2], the last factor in Eq. (13) is approximately 1. Thus, we get:

$$1 \approx \frac{\Gamma(\overline{B^0} \to \pi^+, \rho^-)}{\Gamma(\overline{B^0} \to \rho^+, \pi^-)} \approx \left(\frac{f_{\rho^-}}{f_{\pi^-}}\right)^2 \left[\frac{F_1^{B\to\pi}(0)}{A_0^{B\to\rho}(0)}\right]^2. \tag{14}$$

The ratio $\mathcal{R} \equiv F_1^{B\to\pi}(0)/A_0^{B\to\rho}(0)$ takes different values according with theoretical approach used to calculate numerical values of the form factors evaluated at $t = 0$. In some quark models, for example in [2, 18], and in [19], $\mathcal{R} > 1$; from lattice QCD calculations [20] and from the light-cone sum rules (LCSR) [21] (Table II of Ref. [17] shows explicitly values of these form factors at zero momentum transfer obtained in LCSR) is obtained $\mathcal{R} < 1$, and if we take the values of $F_1^{B\to\pi}(0)$ and $A_0^{B\to\rho}(0)$ reported in Table IV of Ref. [22], which are calculated in LCSR [21] and in lattice-QCD [20], respectively, we obtain $\mathcal{R} \approx 1$. Thus the experimental value of $\mathcal{R}$ can discriminate among these different theoretical approaches[4] to evaluate the form factors at $t = 0$. If we take the numerical values of decay constants reported in [16]: $f_{\pi^-} = 0.1307$ GeV and $f_{\rho^-} = 0.209$ GeV, we obtain from Eq. (14):

$$\mathcal{R} = \left[\frac{F_1^{B\to\pi}(0)}{A_0^{B\to\rho}(0)}\right] \simeq 0.632. \tag{15}$$

4.2 Now let us compare decay widths of $B \to P, P'$ and $B \to P, V$, where P' and V have the same quark content. From expressions in Table III and using monopolar form factors (see appendix B) with the fact that $F_0^{B\to P}(0) = F_1^{B\to P}(0)$, we obtain:

$$\frac{\Gamma(B \to P, P')}{\Gamma(B \to P, V)} = \left(\frac{f_{P'}}{f_V}\right)^2 \left[\frac{1 - m_V^2/m_{1^-}^2}{1 - m_{P'}^2/m_{0^+}^2}\right]^2 \frac{\left[\lambda(m_B^2, m_P^2, m_{P'}^2)\right]^{1/2}}{\left[\lambda(m_B^2, m_P^2, m_V^2)\right]^{3/2}} (m_B^2 - m_P^2)^2. \tag{16}$$

[3] Although we know that the $\rho^-\pi^+$ decay has a larger rate than the $\rho^+\pi^-$ mode mainly because of the difference of the decay constants f_ρ and f_π [17].

[4] See for example Refs. [23, 24] for a summary about values of form factors at zero momentum transfer ($t = 0$) in different models.

Let us considerer the exclusive $\overline{B^0} \to \pi^+, \pi^-$ and $\overline{B^0} \to \pi^+, \rho^-$ decays as an application of Eq. (16). In this case we get:

$$\left[\frac{1 - m_\rho^2/m_{1^-}^2}{1 - m_\pi^2/m_{0^+}^2}\right]^2 \frac{\left[\lambda(m_{B^0}^2, m_\pi^2, m_\pi^2)\right]^{1/2}}{\left[\lambda(m_{B^0}^2, m_\pi^2, m_\rho^2)\right]^{3/2}} (m_{B^0}^2 - m_\pi^2)^2 = 1.067 \approx 1, \tag{17}$$

thus,

$$\frac{\Gamma(\overline{B^0} \to \pi^+, \pi^-)}{\Gamma(\overline{B^0} \to \pi^+, \rho^-)} \simeq \left(\frac{f_{\pi^-}}{f_{\rho^-}}\right)^2. \tag{18}$$

This ratio provides only information about decay constants f_π and f_ρ and it is independent of form factors. Using numerical values of [16] $\mathcal{B}(\overline{B^0} \to \pi^+\pi^-) = 4.6\text{x}10^{-6}$ and $(f_\pi/f_\rho) = (0.1307/0.209) = 0.632$, we obtain $\mathcal{B}(\overline{B^0} \to \pi^+\rho^-) = 1.15\text{x}10^{-5}$. This value is approximately 50.43% of $\mathcal{B}(\overline{B^0} \to \rho^\pm\pi^\mp)$ reported in [16] for the sum of $\rho^+\pi^-$ and $\rho^-\pi^+$ states.

We also can apply Eq. (16) to decays $B^0 \to D^-, \pi^+$ and $B^0 \to D^-, \rho^+$ which branching ratios are $3.4\text{x}10^{-3}$ and $7.5\text{x}10^{-3}$, respectively [16]. Following a similar procedure in last case, and taking numerical values of masses of respective mesons reported in [16] and pole masses given in Ref. [2] we obtain $(f_{\pi^+}/f_{\rho^+}) = 0.651$.

4.3 Another ratio can be obtained comparing decay widths of $B \to P, V$ and $B \to P, A$, where V and A have the same flavor quantum numbers. From expressions in Table III and using monopolar form factors (see appendix B) we obtain:

$$\frac{\Gamma(B \to P, V)}{\Gamma(B \to P, A)} = \left(\frac{f_V}{f_A}\right)^2 \left[\frac{1 - m_A^2/m_{1^-}^2}{1 - m_V^2/m_{1^-}^2}\right]^2 \left[\frac{\lambda(m_B^2, m_P^2, m_V^2)}{\lambda(m_B^2, m_P^2, m_A^2)}\right]^{3/2}. \tag{19}$$

We can apply last equation to $B^0 \to D^-, \rho^+$ and $B^0 \to D^-, a_1^+$ decays. From Ref. [16] we have $\mathcal{B}(B^0 \to D^-\rho^+) = 7.5\text{x}10^{-3}$ and $\mathcal{B}(B^0 \to D^-a_1^+) = 6\text{x}10^{-3}$. Using these branchings, numerical values for the respective masses given in [16] and the 1^--pole mass reported in [2], we obtain from Eq. (19) that $(f_\rho/f_{a_1}) = 1.06$. With $f_\rho = 0.209$ GeV [16] it is obtained $f_{a_1} = 0.197$ GeV. This value is smaller than the value reported in the literature. For example, Ref. [4] gives $f_{a_1} = 0.229$ GeV (extracted from hadronic τ decay $\tau^- \to M^-\nu_\tau$) and $f_{a_1} = 0.256$ GeV (comparing branching ratios of $\overline{B^0} \to D^{*+}, a_1^-$ and $\overline{B^0} \to D^{*+}, \rho^-$ decays). On the other hand, Ref. [25] obtained $f_{a_1} = 0.215$ GeV for $\theta = 32^\circ$ and $f_{a_1} = 0.223$ GeV for $\theta = 58^\circ$, where θ is the mixing angle between K_{1A} and K_{1B} mesons.

We can also relate $B^0 \to \pi^-, \rho^+$ and $B^0 \to \pi^-, a_1^+$ decays. Taking $\mathcal{B}(B^0 \to \pi^-\rho^+) = 1.13\text{x}10^{-5}$, $(f_\rho/f_{a_1}) = 1.06$ (obtained from the before example), the respective numerical values of masses [16] and the respective pole mass [2], we obtain from Eq. (19):

$$\mathcal{B}(B^0 \to \pi^- a_1^+) = 9.7\text{x}10^{-6}. \tag{20}$$

This prediction satisfies the upper bound $\mathcal{B}(B^0 \to \pi^\pm a_1^\mp) < 4.9\text{x}10^{-4}$ reported in [16].

4.4 Let us compare $B \to V, V'$ and $B \to V, A$, where V' and A have the same quark content. From Table III we obtained

$$\frac{\Gamma(B \to V, V')}{\Gamma(B \to V, A)} = \left(\frac{f_{V'}}{f_A}\right)^2 \frac{\mathcal{G}(m_{V'}^2)}{\mathcal{G}(m_A^2)}. \tag{21}$$

Taking the branching ratios $\mathcal{B}(B^0 \to D^{*-}\rho^+) = 6.8\text{x}10^{-3}$ and $\mathcal{B}(B^0 \to D^{*-}a_1^+) = 1.3\text{x}10^{-2}$ [16] and evaluating $\mathcal{G}$ using appropriate monopolar form factors [2], we obtained from Eq. (21) that $(f_\rho/f_{a_1}) = 0.81$, which agrees with the value reported in Ref. [4]. We point out that if we use this value in Eq. (19), now we obtain $\mathcal{B}(B^0 \to \pi^- a_1^+) = 1.65\text{x}10^{-5}$. This prediction is larger than the prediction given in last section (see Eq. (20)) and it is consistent with experimental upper bound [16] for this process. Moreover this prediction is the same order of experimental average value $\mathcal{B}(\overline{B^0} \to \pi^\mp a_1^\pm) = (4.09 \pm 0.76)\text{x}10^{-5}$ [24], which includes BABAR and Belle results, and theoretical predictions obtained in [10, 24].

4.5 Now, we are going to compare expressions of decay rate of *type-I* nonleptonic $B \to M_1, M_2$ channel with differential decay rate of semileptonic $B \to M_1 l\nu_l$ process evaluated in $t = m_{M_2}^2$ at tree level. It is well known that the ratio $R = \Gamma(B \to M_1, M_2)/[d\Gamma(B \to M_1 l\nu_l)/dt|_{t=m_{M_2}^2}]$ provides a method to test factorization hypothesis and may be used to determine some unknown decay constants [4, 26]. Also it is possible combining exclusive semileptonic and hadronic B decays to measure CKM matrix elements (see for example the paper of J. M. Soares in Ref. [7]).

We obtain from Tables II and III, assuming that M_2 is a vector meson and that M_1 is a $l = 0$ or $l = 1$ meson:

$$R = \frac{\Gamma(B \to M_1, V)}{d\Gamma(B \to M_1 l\nu_l)/dt|_{t=m_V^2}} = \frac{\xi^{(V)}}{\zeta} = 6\pi^2 |V_{ij}|^2 a_1^2 f_V^2, \tag{22}$$

where M_1 can be a pseudoscalar (P) or a vector meson (V) or a scalar (S) or an axial-vector (A) or a tensor (T) meson. V_{ij} is the appropriate CKM matrix element (depending on the flavor quantum numbers of meson V), f_V is the decay constant of meson V, and a_1 is the QCD parameter. In general, $R = 6\pi^2|V_{ij}|^2 a_1^2 f_V^2 X_{M_2}^{(*)}$. So, for $M_2 = V$, one has exactly $X_V = X_V^* = X_V^{**} = 1$ (X_V^{**} corresponds to the case when M_1 is a *p-wave* meson: scalar (S), or axial-vector (A) or a tensor (T) meson). Thus, the ratio R, which is model-independent, is a clean and direct test of factorization hypothesis. On the other hand, assuming the validity of the factorization with a fixed value for a_1, it provides an alternative use: it may be employed for determination of unknown decay constants.

Now let us to compare the decay width of $B \to P_1, P_2$ with the differential semileptonic decay rate of $B \to P_1 l\nu_l$ ($m_l \approx 0$). From Tables II and III, it is obtained:

$$\frac{\Gamma(B \to P_1, P_2)}{d\Gamma(B \to P_1 l\nu_l)/dt|_{t=m_2^2}} = \frac{\xi^{(P_2)}}{\zeta} \frac{(m_B^2 - m_1^2)^2}{\lambda(m_B^2, m_1^2, m_2^2)} \left[\frac{1 - m_2^2/m_{1^-}^2}{1 - m_2^2/m_{0^+}^2}\right]^2. \tag{23}$$

If $m_1,\ m_2 \ll m_B$, $(m_B^2 - m_1^2)^2/\lambda \approx 1$. This condition also implies that $m_2 \ll m_{1^-}, m_{0^+}$ and the last factor in Eq. (23) is of the order 1. From these conditions, we get:

$$\frac{\Gamma(B \to P_1, P_2)}{d\Gamma(B \to P_1 l\nu_l)/dt|_{t=m_2^2}} \approx \frac{\xi^{(P_2)}}{\zeta} = 6\pi^2 |V_{q_i q_j}|^2 a_1^2 f_{P_2}^2. \tag{24}$$

As an application of Eq. (24) we can compare $B \to \pi, \pi$ and $B \to \pi l \nu_l$ decays.

Finally, let us mention that there are several ratios, combining last equations, that can be of some interest. For example:

$$\frac{\Gamma(B \to P, V)}{\Gamma(B \to V_1, V)} = \frac{d\Gamma(B \to P l \nu_l)/dt|_{t=m_V^2}}{d\Gamma(B \to V_1 l \nu_l)/dt|_{t=m_V^2}}, \tag{25}$$

and

$$\frac{\Gamma(B \to P, P_2)}{d\Gamma(B \to P l \nu)/dt|_{t=m_2^2}} \frac{d\Gamma(B \to P l \nu)/dt|_{t=m_V^2}}{\Gamma(B \to P, V)} = \left(\frac{f_{P_2}}{f_V}\right)^2, \tag{26}$$

where P_2 and V mesons have the same flavor quantum numbers.

5 Concluding remarks

We have performed a brief analysis about expressions for $\Gamma(B \to M_1, M_2)$ and $d\Gamma(B \to M_1 l \nu)/dt$ at tree level including all the mesons $l = 0$ and $l = 1$ in final state. Indeed, we have considered that M_1 and M_2 can be a pseudoscalar (P), a vector (V), a scalar (S), an axial-vector (A) or a tensor (T) meson. We have assumed factorization hypothesis and used the parametrizations of $\langle M|J_\mu|B\rangle$ given in WSB and ISGW quark models. We explain some aspects related with dynamics of these processes and give some useful ratios between decay widths that can determine some form factors, decay constants and branching ratios.

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Appendix A: Relations between form factors of WSB and ISGW quark models.

We can obtain form factors of WSB model [2] in function of form factors of ISGW model [3] comparing the parametrizations given in both models for $B \to P$ and $B \to V$ transitions. Thus, from $\langle P|J_\mu|B\rangle_{WSB} = \langle P|J_\mu|B\rangle_{ISGW}$ we obtain the following relations:

$$F_0(t) = \frac{t}{(m_B^2 - m_1^2)} \, (f_+(t) + f_-(t)), \tag{27}$$

$$F_1(t) = f_+(t), \tag{28}$$

and from $\langle V|J_\mu|B\rangle_{WSB} = \langle V|J_\mu|B\rangle_{ISGW}$ it is obtained:

$$A_0(t) = \frac{1}{2m_V} \left[f(t) + t a_-(t) + (m_B^2 - m_V^2) a_+(t) \right], \tag{29}$$

$$A_1(t) = \frac{f(t)}{(m_B + m_V)}, \tag{30}$$

$$A_2(t) = -(m_B + m_V)\, a_+(t), \tag{31}$$

$$V(t) = -(m_B + m_V)\, g(t). \tag{32}$$

Using these relations it is straightforward to get $d\Gamma(B \to P(V)l\nu)/dt$ or $\Gamma(B \to P(V), M)$ in one model from the respective expressions in the other model.

Appendix B: Form factors in WSB quark model.

In this section we show the expressions for monopolar form factors that appear in the parametrizations of $\langle P|J_\mu|B\rangle$ and $\langle V|J_\mu|B\rangle$ in WSB model (see Refs. [2] and [5]):

$$F_0(t) = \frac{F_0(0)}{1 - t/m_{0^+}^2}, \tag{33}$$

$$F_1(t) = \frac{F_1(0)}{1 - t/m_{1^-}^2}, \tag{34}$$

$$A_0(t) = \frac{A_0(0)}{1 - t/m_{0^-}^2}, \tag{35}$$

$$A_i(t) = \frac{A_i(0)}{1 - t/m_{1^+}^2}, \qquad i = 1, 2, 3. \tag{36}$$

$$V(t) = \frac{V(0)}{1 - t/m_{1^-}^2}. \tag{37}$$

In section 3 we explain how to obtain quantum numbers J^P for poles.

The helicity form factors are [2]:

$$H_\pm(t) = (m_B + m_V)A_1(t) \mp \frac{2m_B \mathcal{Z}}{(m_B + m_1)} V(t), \tag{38}$$

$$H_0(t) = \frac{1}{2m_1\sqrt{t}} \left[(m_B^2 - m_1^2 - t)(m_B + m_1)A_1(t) - \frac{4m_B^2 \mathcal{Z}^2}{(m_B + m_1)} A_2(t) \right], \tag{39}$$

where $\mathcal{Z} = 1/2m_B \left[(m_B^2 - m_1^2 - t)^2 - 4m_1^2 t \right]^{1/2}$.

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